EE 565 Mini-Project 2, Winter 2008

Analysis and Simulation of Queueing Systems

Submit Online by 10:30AM on 25 February 2008

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1 Project Description

The goal of this project is to analyze and simulate a queueing network, an interconnected system of queues. You will be responsible for writing the simulation code for users arriving randomly into the queue and moving between nodes (reusing code from your previous project is recommended), as well as the tools for analyzing the network state. You may use any (reasonable) programming language (MATLAB is recommended). Prepare a short (1-3 page) project write-up containing the following:

- A brief summary of the project,
- Any assumptions you had to make (e.g. parameters, unspecified quantities),
- A summary of your results and figures, and
- A description of your implementation.

Create a .zip, .tar, or .rar archive including (1) your project write-up, (2) your source code, and (3) a text file with instructions to run your code. Submit this archive according to the online submission instructions.

2 Analysis and Implementation of Queues

Q1: Implement each of the $M/M/1$, $M/M/k$, and $M/M/k/m$ ($k \geq m$) queues with Poisson arrival and service processes with rates λ and μ, respectively. For each queue, compare the analytical and empirical values of:

1. The steady-state distribution π of the total number of customers,
2. The probability that an arriving customer is turned away from a queue (in the case of $M/M/k/m$),
3. The expected queueing delay T and waiting time W,
4. The utilization ρ (expected number of busy servers)

for several appropriate values of λ, μ, m, and k, including cases in which $\lambda \geq \mu$. Discuss your results and any interesting observations from your simulation study.

3 Queueing Networks

A queueing network of size N is a collection of N independent interconnected queues. The i^{th} queue is an $M/M/k_i$ queue with servers of rate μ_i and arrivals from outside the network with rate γ_i. Upon receiving service from queue i, a customer moves to queue j with probability r_{ij} for

$j = 1, \ldots, N$ and leaves the network with probability $1 - \sum_{j=1}^{N} r_{ij}$. Letting λ_i^* denote the departure rate from node i, we see that the total arrival rate into queue i is given by

$$\lambda_i = \gamma_i + \sum_{j=1}^{N} \lambda_j^* r_{ij}.$$

Let R denote the N-by-N matrix of probabilities r_{ij}.

Q2: Consider a wireless network of N nodes uniformly distributed over a region $\mathcal{A}$ such that each node can communicate within radio range r. Assume that $\pi r^2 \ll |\mathcal{A}|$ (i.e. large-scale network) and that $N/|\mathcal{A}| \gg 1$ (i.e. dense network). Each node generates data packets at a rate λ_{gen} to be routed to a random node in the network. Each time a node i has a packet to forward to a final destination j: (1) if j is within range r, i sends directly to j, otherwise (2) i chooses the next-hop node randomly from the set of neighbors which are closer to j with probabilities weighted according to the distance from the destination node (i.e. higher weight to those closer to the destination). Ignoring interference/collisions, investigate the impact of the parameters λ_{gen}, N, and r on the average total queueing delay experienced by each routed packet. How does the average delay relate to the distance between the source node and destination node?

4 Closed Queueing Networks

A closed queueing network is a queueing network in which there are neither arrivals into nor departures from the system. In such a system, $\gamma_i = 0$ for all nodes i and each row of the matrix R must sum to 1. Since customers are never added to or removed from the system, the total network population K remains constant.

Q3: Consider the following example. Suppose that K UW students independently move between four locations: UW campus, a restaurant on University Way, a favorite pub in Wallingford, and home. Suppose that the UW acts as an $M/M/\infty$ queue with service rate $\mu_{\text{uw}} = 1/8$ customers/hour, the restaurant acts as an $M/M/20$ queue with service rate $\mu_{\text{ave}} = 2$ customers/hour, the pub acts as an $M/M/50$ queue with service rate $\mu_{\text{pub}} = 1/2$ customers/hour, and home acts as an $M/M/\infty$ queue with service rate $\mu_{\text{home}} = 1/8$ customers/hour. In addition, suppose the transition matrix R (with rows and columns ordered as above) is given by

$$R = \begin{bmatrix} 0 & 0.4 & 0.2 & 0.4 \\ 0.3 & 0 & 0.6 & 0.1 \\ 0.1 & 0.2 & 0 & 0.7 \\ 0.7 & 0.2 & 0.1 & 0 \end{bmatrix}.$$

Implement a closed queueing network for the above example for several values of K ranging from 10 to 1000.

1. Evaluate the average fraction of time each student spends in each of the four locations. What tendencies do you notice? How are these tendencies affected by changes in the service rates? changes in the transition probabilities?
2. Treating the transition matrix R as that of a discrete-time Markov chain X, find the stationary distribution π of X. How does this relate to the answer to the previous part?